

## Keywords, statements, definitions

Let  $p$  be a prime.

1. Proof of the final statement of Sylow's theorem: If  $G$  is a finite group. Let  $Q \leq G$  be a  $p$ -group. Then  $Q$  is contained in a Sylow  $p$ -subgroup of  $G$ .
2. The center of a finite  $p$ -group is nontrivial.
3. Let  $P$  be a finite  $p$ -group of order  $p^a$ . Assume  $0 \leq b \leq a$ . Then  $P$  has a (normal) subgroup of order  $p^b$ .
4. Corollary of the second statement: Every group of order  $p^2$  is Abelian. There are two nonisomorphic:  $C_{p^2}$  and  $C_p \times C_p$ .
- 5.

**Definition 0.1.** *Frobenius groups: Transitive permutation group  $G$  on a finite set  $X$ . The intersection of the stabilizer of any 2 different elements of  $X$  is  $\{1\}$ . The stabilizer of any element is nontrivial.*

We will prove at the end of the semester that

$$K = \{1\} \cup \{\text{fixed point free elements}\}$$

is a normal subgroup of  $G$ .

- 6.

**Definition 0.2.**  $G \leq S_n$  is called a  $k$ -transitive permutation group if for every  $1 \leq a_1 < a_2 < \dots < a_k$  and every  $k$  different elements of  $\{1, \dots, n\}$  there is an element  $g \in G$  such that  $a_i^g = b_i$